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Three-dimensional instability of two nonlinearly coupled electromagnetic waves in a plasma

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Abstract

The three-dimensional instability of two coupled electromagnetic waves in an unmagnetized plasma is investigated theoretically and numerically. In the regime of two-plasmon decay, where one pump wave frequency is approximately twice the electron plasma frequency, we find that the coupled pump waves give rise to enhanced instability with wave vectors between those of the two beams. In the case of ion parametric decay instability, where the pump wave decays into one Langmuir wave and one ion acoustic wave, the instability regions are added with no distinct amplification. Our investigation can be useful in interpreting laser-plasma as well as ionospheric heating experiments.

I. INTRODUCTION

The development of high-power lasers has made it possible to study the relativistic regime of laser-plasma interactions. This is of fundamental importance for laser-induced heating and inertial confinement fusion [1] and poses also the possibility to test fundamental physics. The first theoretical studies of relativistic waves in plasmas date back to the fifties [2], where it was recognized that the the quivering velocity of electrons may lead to relativistic mass increase in an ultra-intense electromagnetic wave. The relativistic effects can result in self-modulation and self-focusing of electromagnetic waves in plasmas [3]. The instability of relativistically large amplitude electromagnetic waves has been studied for magnetized electron plasmas [4], electron-ion plasmas [5, 6], electron beam plasma systems [7], hot electron-ion plasmas [8], and magnetized hot plasmas [9, 10]. The instability of relativistically strong laser light in an unmagnetized plasma has then been revisited [11]. The presence of multiple laser beams in a plasma can give rise to a new set of interesting phenomena [12, 13, 14, 15]. An important application of two interacting laser beams in plasmas is the excitation of large amplitude Langmuir waves, which in turn accelerate electrons to ultra-relativistic speeds [13]. The nonlinear coupling between two electromagnetic waves in plasmas can be described by a system of coupled nonlinear Schrödinger equations that model nonlinear interactions between localized light [15, 16] and Langmuir or ion-acoustic waves. At strongly relativistic intensities, laser beams can give rise to fast plasma waves via higher-order nonlinearities [12, 13, 17], or via the beat wave excitation at frequencies different from the electron plasma frequency [18]. Particle-in-cell simulations [19] show that large-amplitude electron plasma waves can be excited by colliding laser pulses, or by two co-propagating electromagnetic pulses where a long trailing pulse is modulated efficiently by the periodic plasma wake behind the heading short laser pulse [20]. The effects on parametric instabilities of a partially incoherent pump wave (with a distribution of wave modes) was investigated both theoretically [21] and experimentally [22], where it was found that the effect of finite bandwidth is, in general, to increase the instability thresholds and lower the growth rate. The nonlinear interaction between two electromagnetic waves in the Earth's ionosphere has also been considered [23, 24, 25]. In that case, it is, in addition, important to focus attention on the collisional coupling between the waves, e.g. [26, 27, 28]. Related phenomena occur also in semiconductor plasmas [29, 30].

Our previous treatment of the instability of two coupled laser beams in an electron-ion plasma [31] and a two-temperature electron plasma [32] were limited to the investigation of the relativistic Raman and Brillouin scattering instabilities in one dimension, and two-dimensional effects were only partly included. The purpose of the present paper is to include the multi-dimensional effects, which are particularly important for the cases where an electromagnetic wave decays into one low-frequency wave and one high-frequency electrostatic wave that are propagating obliquely to the pump wave.

II. MODEL EQUATIONS

Let us consider the propagation of intense laser light in an electron–ion plasma. The dynamics of the high frequency laser light is governed by

$$\frac{\partial^2 \mathbf{A}_h}{\partial t^2} + c^2\nabla\times(\nabla\times\mathbf{A}_h) - 3v_{Te}^2\nabla(\nabla\cdot\mathbf{A}_h) + \omega_{pe}^2(1+N_{es})\mathbf{A}_h - \frac{\omega_{pe}^2 e^2}{m_e^2 c^4}\langle|\mathbf{A}_h|^2\rangle\mathbf{A}_h = 0, \quad (1)$$

where $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, n_0 is the equilibrium number density, e is the magnitude of the electron charge, m_e is the electron mass, v_{Te} is the electron thermal speed, c is the speed of light, and the normalized slow time-scale electron number density perturbation is $N_{es} = n_{es}/n_0$. The latter is excited by the ponderomotive force of the high frequency waves. If the ions are considered as immobile, we have [31]

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_{Te}^2\nabla^2\right) N_{es} = \frac{e^2}{m_e^2 c^2}\nabla^2\langle|\mathbf{A}_h|^2\rangle \quad (2)$$

whereas if the electrons are treated as inertialess [31]

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\nabla^2\right) N_{es} = \frac{e^2}{m_e m_i c^2}\nabla^2\langle|\mathbf{A}_h|^2\rangle \quad (3)$$

where c_s is the sound speed.

We will here consider two large amplitude electromagnetic waves $\mathbf{A}_h = \mathbf{A}_1 + \mathbf{A}_2$. We then have $|\mathbf{A}_h|^2 = |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2\mathbf{A}_1\cdot\mathbf{A}_2$.

Using equations (1)–(3) we can now derive a nonlinear dispersion relation. The calculations are straightforward but lengthy. Following closely the analysis of Ref. [33], it then turns out to be convenient to introduce a characteristic velocity v_{tS} that is defined by

$$v_{tS,j}^2 = \omega_{pe}^2 \sum_{+,-}\left[\frac{|\mathbf{k}_{j\pm}\times\mathbf{v}_{0j}|^2}{k_{j\pm}^2 D_{j\pm}} + \frac{|\mathbf{k}_{j\pm}\cdot\mathbf{v}_{0j}|^2}{k_{j\pm}^2\omega_{j\pm}^2\varepsilon_{j\pm}}\right] \quad (4)$$

where $\omega_{j\pm} = \omega \pm \omega_{0j}$, $\mathbf{k}_{j\pm} = \mathbf{k} \pm \mathbf{k}_{0j}$, ω_{0j} and $\mathbf{k}_{0j}$ is the pump frequency and wavevector of the pump wave j ($j = 1, 2$), whereas ω and $\mathbf{k}$ are associated with the electrostatic low-frequency fluctuations. Furthermore, we have here introduced the pump wave quiver velocity $\mathbf{v}_{0j} = e\mathbf{A}_{0j}/m_e c$, $D_{j\pm} = \omega_{j\pm}^2 - \omega_{pe}^2 - k_{j\pm}^2 c^2 + i\omega_{j\pm}\gamma_{j\pm}$ where $\gamma_{j\pm}$ represents the sideband damping [33], and $\omega_{j\pm}^2 \varepsilon_{j\pm} \approx \omega_{j\pm}^2 - \omega_{pe}^2 - 3k_{j\pm}^2 v_{Te}^2$. For the pump frequencies, we use $\omega_{0j}^2 = \omega_{pe}^2 + k_{0j}^2 c^2$.

In the present paper we will, for simplicity, further assume that $\mathbf{A}_1 \cdot \mathbf{A}_2 \approx 0$. This means that double resonance parametric phenomena [34], where the difference between the frequencies ω_{01} and ω_{02} is close to twice a natural frequency, will be neglected. Choosing two transverse waves that propagate in the $y-$ and z-directions, respectively with the pump velocities in the $z-$ and y-directions, respectively, we note that (4) reduces to

$$v_{tS,j}^2 \approx \frac{\omega_{pe}^2 e^2}{m_e^2 c^2}\left[\frac{|\mathbf{k}_{j+} \times \mathbf{A}_{0j}|^2}{k_{j+}^2 D_{j+}} + \frac{|\mathbf{k}_{j-} \times \mathbf{A}_{0j}|^2}{k_{j-}^2 D_{j-}} + \frac{|\mathbf{k} \cdot \mathbf{A}_{0j}|^2}{k_{j+}^2 \omega_{j+}^2 \varepsilon_{j+}} + \frac{|\mathbf{k} \cdot \mathbf{A}_{0j}|^2}{k_{j-}^2 \omega_{j-}^2 \varepsilon_{j-}}\right] \tag{5}$$

With these limitations, and following Refs. [31] and [33], the nonlinear dispersion relation turns out to be

$$\begin{aligned}\frac{1}{Q} &+ \frac{|\mathbf{k}_{1+} \times \mathbf{A}_{01}|^2}{k_{1+}^2 D_{1+}} + \frac{|\mathbf{k}_{1-} \times \mathbf{A}_{01}|^2}{k_{1-}^2 D_{1-}} + \frac{|\mathbf{k}_{2+} \times \mathbf{A}_{02}|^2}{k_{2+}^2 D_{2+}} + \frac{|\mathbf{k}_{2-} \times \mathbf{A}_{02}|^2}{k_{2-}^2 D_{2-}} \\ &+ \frac{|\mathbf{k} \cdot \mathbf{A}_{01}|^2}{k_{1+}^2 \omega_{1+}^2 \varepsilon_{1+}} + \frac{|\mathbf{k} \cdot \mathbf{A}_{01}|^2}{k_{1-}^2 \omega_{1-}^2 \varepsilon_{1-}} + \frac{|\mathbf{k} \cdot \mathbf{A}_{02}|^2}{k_{2+}^2 \omega_{2+}^2 \varepsilon_{2+}} + \frac{|\mathbf{k} \cdot \mathbf{A}_{02}|^2}{k_{2-}^2 \omega_{2-}^2 \varepsilon_{2-}} = 0.\end{aligned} \tag{6}$$

In Eq. (6), as well as below, we have normalized $\mathbf{A}_{0j}$ by $m_e c^2/e$. For electron Langmuir waves, we have [31]

$$Q_L = \omega_{pe}^2 \left(1 - \frac{k^2 c^2}{\omega^2 - 3k^2 v_{Te}^2 - \omega_{pe}^2}\right). \tag{7}$$

whereas for ion acoustic waves [31]

$$Q_{IA} = \omega_{pe}^2 \left(1 - \frac{m_e}{m_i} \frac{k^2 c^2}{(\omega^2 - k^2 c_s^2)}\right). \tag{8}$$

III. RESULTS

We have carried out a numerical analysis of Eq. (6), where we have assumed that the frequency is complex valued, where the imaginary part of ω represents the growth rate. In our treatment, we have concentrated on three-wave decay processes where the important terms in Eq. (6) are the ones with down-shifted daughter waves. Hence, we have kept the

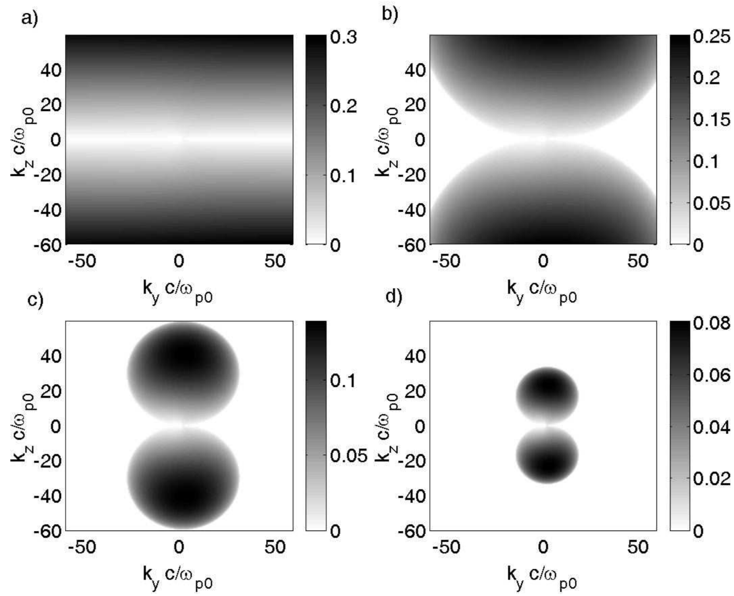

FIG. 1: The growth rate (normalized by ω_{pe}) of the two-plasmon decay of one laser beam for different electron thermal speeds a) $v_{Te} = 0$, b) $v_{Te} = 0.005\,$c, c) $v_{Te} = 0.0075\,$c, and d) $v_{Te} = 0.01\,$c. The pump amplitude is $\mathbf{A}_{01} = 0.01\,\widehat{\mathbf{z}}$, the pump wavevector $\mathbf{k}_{01} = \sqrt{3}\widehat{\mathbf{y}}\omega_{pe}/$c. The second laser beam intensity is set to zero ($\mathbf{A}_{02} = \mathbf{0}$).

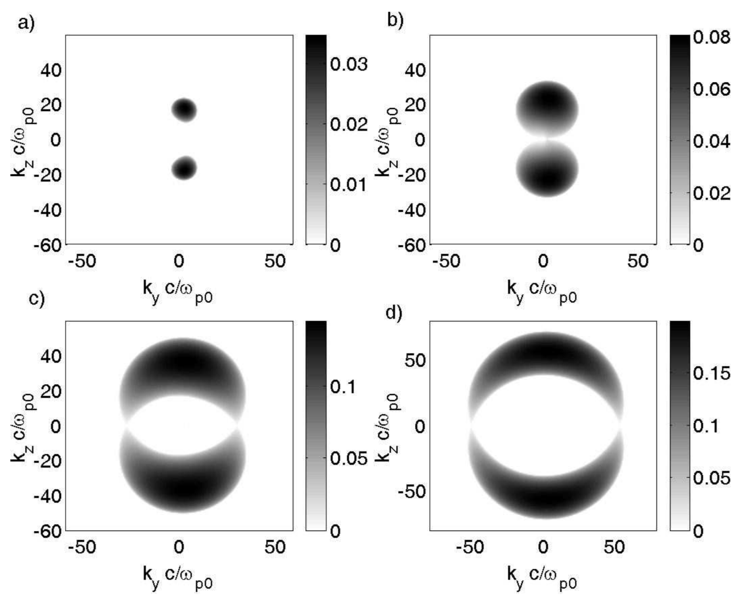

FIG. 2: The growth rate (normalized by ω_{pe}) of the two-plasmon decay of one laser beam for wavevectors a) $\mathbf{k}_{01} = 1.65\widehat{\mathbf{y}}\omega_{pe}/$c, b) $\mathbf{k}_{01} = \sqrt{3}\widehat{\mathbf{y}}\omega_{pe}/$c, c) $\mathbf{k}_{01} = 2\widehat{\mathbf{y}}\omega_{pe}/$c, and d) $\mathbf{k}_{01} = 2.5\widehat{\mathbf{y}}\omega_{pe}/$c. The pump amplitude is $\mathbf{A}_{01} = 0.01\,\widehat{\mathbf{z}}$, and the electron thermal speed $v_{Te} = 0.01\,$c. The second laser beam intensity is set to zero ($\mathbf{A}_{02} = \mathbf{0}$).

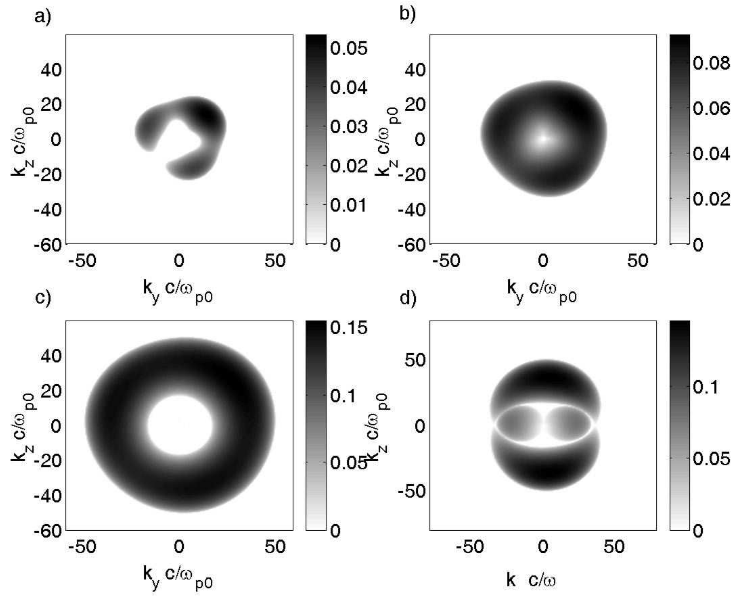

FIG. 3: The growth rate (normalized by ω_{pe}) of the two-plasmon decay of two coupled laser beams for wavevectors a) $\mathbf{k}_{01} = 1.65\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = 1.65\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$, b) $\mathbf{k}_{01} = \sqrt{3}\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = \sqrt{3}\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$, c) $\mathbf{k}_{01} = 2\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = 2\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$, and d) $\mathbf{k}_{01} = 2\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = \sqrt{3}\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$. The pump amplitudes are $\mathbf{A}_{01} = 0.01\,\widehat{\mathbf{z}}$ and $\mathbf{A}_{02} = 0.01\,\widehat{\mathbf{y}}$, and the electron thermal speed is $v_{Te} = 0.01\,\mathrm{c}$.

resonant terms with subscripts "-" in Eq. (6) but neglected those with subscripts "+". For large wavenumbers, we recover almost identically the stimulated Raman and Brillouin cases treated in Ref. [31]. Thus, we shall here concentrate on the regimes of smaller wavenumbers in which the two-plasmon decay and ion parametric decay instabilities become important, and where it is crucial to have a fully multi-dimensional treatment.

We first consider the two-plasmon case in which the low-frequency wave is a Langmuir wave, and where Q in Eq. (6) equals Q_L in (7). The growth rates (normalized by ω_{pe}) are presented in Figs. 1–3. We have here denoted the unit vectors in the y and z directions by $\widehat{\mathbf{y}}$ and $\widehat{\mathbf{z}}$. In Fig. 1, the electron thermal effects are investigated. The dispersion of the Langmuir waves due to the electron pressure causes the instability to restricted in a bounded domain in wavevector space, with a maximum growth rate in a direction perpendicular to the propagation direction of the laser beam. In Fig. 2, we have investigated how the instability depends on the pump wavenumber of a single electromagnetic beam. We find that the instability is dominant for wavenumbers equal to or larger than $\sqrt{3}\omega_{pe}/c$, and that the

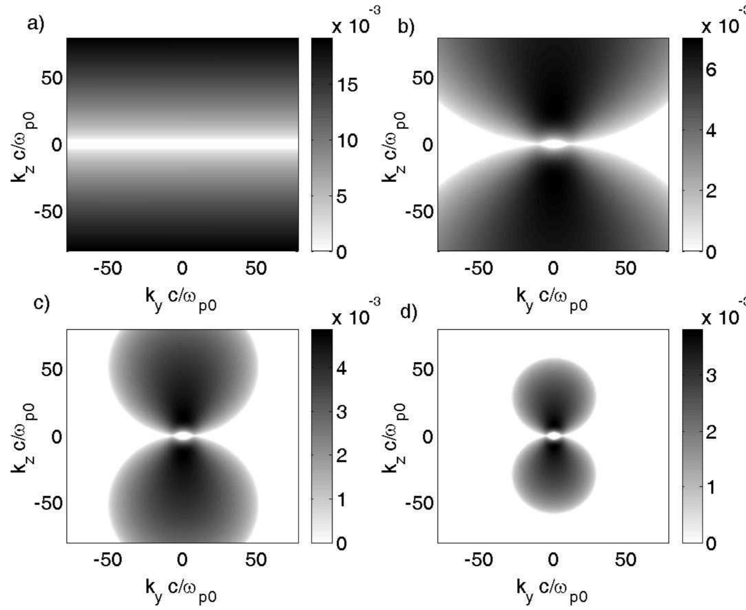

FIG. 4: The growth rate (normalized by ω_{pe}) of the ion parametric decay instability of one laser beam for different electron thermal speeds a) $v_{Te} = 0$, b) $v_{Te} = 0.005\,\mathrm{c}$, c) $v_{Te} = 0.0075\,\mathrm{c}$, and d) $v_{Te} = 0.01\,\mathrm{c}$; in each case the ion acoustic speed is set to $c_s = 0.006\,v_{Te}$. The pump amplitude is $\mathbf{A}_{01} = 0.01\,\widehat{\mathbf{z}}$, and the pump wavevector is $\mathbf{k}_{01} = 0.1\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$. The second laser beam intensity is set to zero ($\mathbf{A}_{02} = \mathbf{0}$).

instability vanishes for smaller wavenumbers of the pump wave. The maximum instability occurs for wavevectors almost perpendicular to the laser beam propagation direction. The instability maximum occurs for wavenumbers much larger than the pump wavenumber. Eventually kinetic effects will become important and electron Landau damping will decrease the growth rate. The case of two coupled electromagnetic beams is investigated in Fig. 3. While beam 1 is directed in the y-direction as in Figs. 1 and 2, beam 2 is here in the z-direction, perpendicular to beam 1. We see in panel a) of Fig. 3 that the coupled beam system gives rise to a new instability with a maximum growth rate in the direction of the dichotome in the center between the two beam propagation directions. For wavenumbers larger than $\sqrt{3}\omega_{pe}/c$, the instability becomes more evenly distributed in all directions, but with a well-defined maximum growth rate for some wavenumber. For the case where the wavenumber of beam 2 is smaller than the one of beam 1, in panel d), we see a superposition of the two instability regions for the separate beams. We note that similar effects can also appear due to direct subharmonic wave generation in nonuniform plasmas [35].

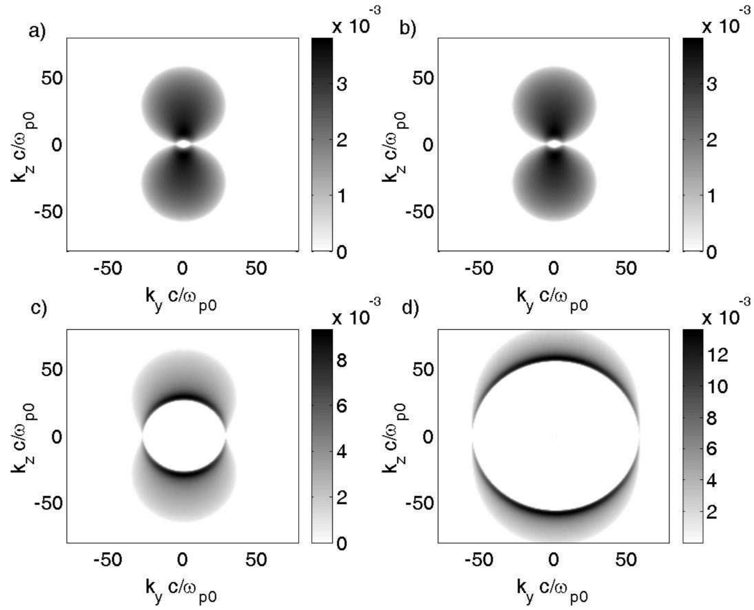

FIG. 5: The growth rate (normalized by ω_{pe}) of the ion parametric decay instability of one laser beam for wavevectors a) $\mathbf{k}_{01} = \mathbf{0}$, b) $\mathbf{k}_{01} = 0.1\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$, c) $\mathbf{k}_{01} = 0.5\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$, and d) $\mathbf{k}_{01} = \widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$. The pump amplitude is $\mathbf{A}_{01} = 0.01\,\widehat{\mathbf{z}}$, the electron thermal speed is $v_{Te} = 0.01\,\mathrm{c}$, and the ion acoustic speed is $c_s = 6 \times 10^{-5}c$. The second laser beam intensity is set to zero ($\mathbf{A}_{02} = \mathbf{0}$).

We next investigate the parametric decay instability in which the electromagnetic wave decays into one electrostatic wave and one low-frequency ion acoustic wave, where Q in Eq. (6) equals Q_{IA} in (8). The growth rates are presented in Figs. (4)–(6). We have here concentrated on the long wavelength limit where the main instability is the ion parametric decay instability, in which the electromagnetic wave decays into one slightly frequency-downshifted electrostatic wave and one low-frequency ion acoustic wave. In Fig. 4, we have studied the thermal effect on the instability of one single electromagnetic beam. We see that for higher electron thermal and ion acoustic speeds, the region of instability becomes smaller in wavenumber space, and that there is a well-defined maximum of the instability for propagation almost perpendicular to the wave propagation direction. In Fig. 5, we have considered different wavenumbers of the pump wave. We see that for large wavenumbers, the maximum instability occurs for larger wavevectors perpendicular to the pump wavevector. Finally, we consider the interaction between two coupled electromagnetic beams in Fig. 6. For the cases of equal pump amplitudes and lengths of the wavevectors, in panels a)–c), the instability becomes almost rotationally symmetric, with equal maximum growth rates in all

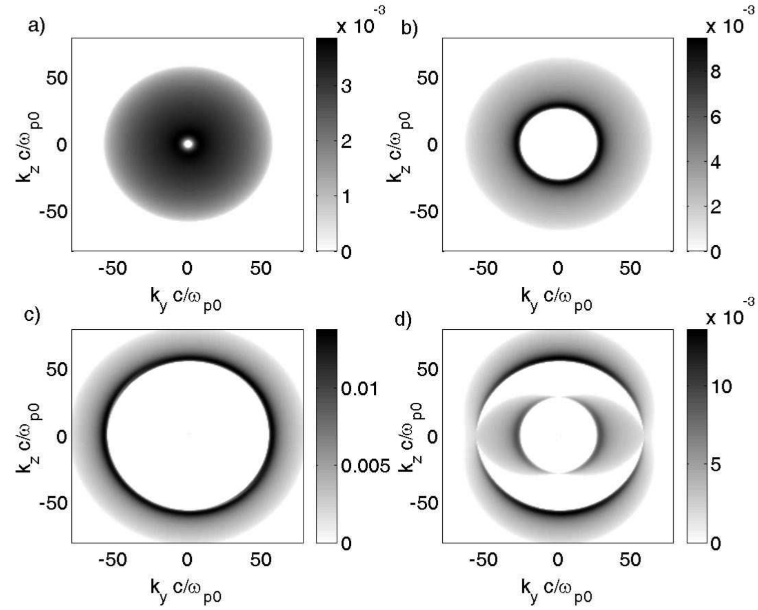

FIG. 6: The growth rate (normalized by ω_{pe}) of the ion parametric decay instability of two coupled laser beams for wavevectors a) $\mathbf{k}_{01} = 0.1\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = 0.1\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$, b) $\mathbf{k}_{01} = 0.5\widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = 0.5\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$, c) $\mathbf{k}_{01} = \widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = \widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$, and d) $\mathbf{k}_{01} = \widehat{\mathbf{y}}\omega_{pe}/\mathrm{c}$ and $\mathbf{k}_{02} = 0.5\widehat{\mathbf{z}}\omega_{pe}/\mathrm{c}$. The pump amplitudes are $\mathbf{A}_{01} = 0.01\,\widehat{\mathbf{z}}$ and $\mathbf{A}_{02} = 0.01\,\widehat{\mathbf{y}}$, the electron thermal speed is $v_{Te} = 0.01\,\mathrm{c}$ and the ion acoustic speed is $c_s = 6 \times 10^{-5}c$.

directions. For the case where the wavenumber of beam 2 is smaller than the one of beam 1, in panel d), we see a superposition of the two instability regions for the separate beams. For the ion parametric decay instability, we could not see the same distinct amplification of the instabilities as we could see for the two-plasmon decay in panel a) of Fig. 3.

In conclusion, we have investigated the instability of two coupled large-amplitude electromagnetic waves in a plasma. Our investigation shows that two-plasmon decay plays an important role for pump frequencies approximately two times larger than the background electron plasma frequency, and that one has a maximum growth rate perpendicular to the propagation direction of the electromagnetic wave. For two coupled electromagnetic beams, there is here a new and stronger instability in a direction between the two laser beams. The ion parametric decay instability, in which an electromagnetic wave decays into one Langmuir wave and one ion acoustic wave, is important for long wavelengths of the electromagnetic pump wave, where the instability leads to ion acoustic waves that are propagating perpendicularly to the electromagnetic beam direction. Here, the addition of a second elec-

tromagnetic beam leads to a superposition of the instabilities. However, we see no strong nonlinear amplification due to the two beams. Our study could be important for both laser-plasma interactions and for ionospheric heating experiments [36, 37, 38, 39].

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